Int. J. Solids Structures Vol. 27, No. 9, pp. 1209–1211, 1991
Printed in Great Britain.

0020-7683/91 \$3.00+ .00
Pergamon Press plc

LETTER TO THE EDITOR

Discussion of "Torsional vibrations of a circular disk on an infinite transversely isotropic medium", *Int. J. Solids Structures*, Vol. 25, No. 9, pp. 1069–1076 (1989)

Tsai has presented a solution to the problem of torsional steady state oscillation of a circular disk bonded to a "transversely isotropic" (also called "cross-anisotropic"), homogeneous elastic half-space. To this end, he has employed Hankel integral transform and contour integration techniques.

The rigorous solution of this problem, as well as of the more complicated boundary value problem of an embedded cylindrical foundation in a transversely isotropic half-space, was presented by us 6 years ago (Constantinou and Gazetas, 1984). To this end, we employed a much simpler approach, which is briefly outlined here.

To solve the governing differntial equation [eqn (3) in Tsai, 1989], we introduced the following transformation:

$$\zeta = \delta z, \tag{40}$$

where δ is given by eqn (5) in Tsai (1989). However, δ^2 is in fact the ratio of the shear modulus in the vertical plane to the shear modulus in the horizontal plane; i.e. $\delta^2 = G_{HH}/G_{VH}$, in the terminology of our paper (Constantinou and Gazetas, 1984). With this transformation, eqn (3) reduces to

$$\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(rU_\theta)\right]+\frac{\partial^2 U_\theta}{\partial \zeta^2}=\frac{2\Delta}{C_{11}-C_{12}}\frac{\partial^2 U_\theta}{\partial t^2}, \tag{41}$$

which is identical in form to the **torsional wave equation in an isotropic continuum**; it describes torsional S-waves propagating in a fictitious transformed space with an apparent velocity:

$$C_A = \left(\frac{C_{11}-C_{12}}{2\Delta}\right)^{1/2} = \delta\left(\frac{C_{44}}{\Delta}\right)^{1/2} = \delta c_2. \tag{42}$$

Hence, the solution for a transversely isotropic medium can be directly reduced from that for an isotropic half-space (Luco and Westman, 1971).

Based on this observation, we express the dynamic torque–rotation relation for a massless circular surface disk as:

$$\frac{M}{\Theta} = K_A[s_A(a_{0A})+ia_{0A}c_A(a_{0A})] \tag{43}$$

in which K_A is the static stiffness, given by

$$K_A = \frac{16}{3}\sqrt{G_{VH}\cdot G_{\mathrm{HH}}}\,a^3 = \frac{16}{3}\delta C_{44}a^3 \tag{44}$$

in our symbols and Tsai's symbols, respectively; a_{0A} is a dimensionless frequency factor:

$$a_{0A} = \frac{\omega a}{C_A} = \frac{k}{\delta}, \tag{45}$$

where k is Tsai's frequency factor. The frequency dependent variables S_A and C_A; represent

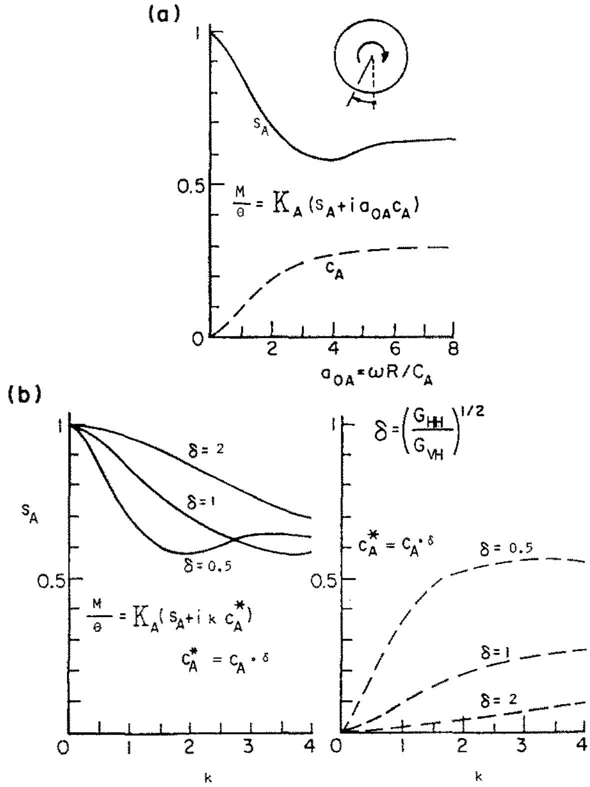

Fig. 4. Torsional dynamic stiffness and damping coefficients for a circular disk on the surface of a transversely isotropic homogeneous half-space, plotted: (a) in terms of a_{0A}; and (b) in terms of k and δ.

the dimensionless dynamic stiffness and damping coefficients, which for a given frequency factor a_{0A} are identical to those obtained for an isotropic half-space with S-wave velocity C_A; hence they are recovered from the aforementioned publication.

Figure 4 portrays the variation of these stiffness and damping coefficients S_A and C_A, first as a **unique** function of a_{0A}, and second as a function of k, with δ as an independent parameter. Notice the wide range of k values in this figure ($0 < k < 4$).

We have applied the same simple transformation to an embedded cylindrical foundation in a transversely isotropic half-space. Again, the solution is recovered from that for an isotropic half-space (Luco, 1976; Day, 1977). The dynamic moment–rotation ratio for a massless foundation is again written in the form of eqn (43), but in this case the static stiffness becomes:

$$K_A = \frac{16}{3}\delta C_{44}a^3\left(\chi + \frac{3\pi}{4}\frac{D}{a}\delta\right); \tag{46}$$

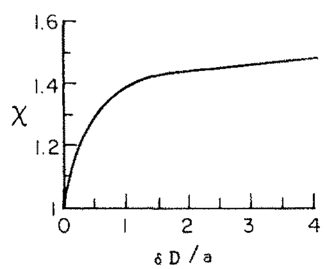

Fig. 5. Factor in eqn (46) for the static torsional stiffness of a cylindrical foundation embedded in a transversely isotropic homogeneous half-space.

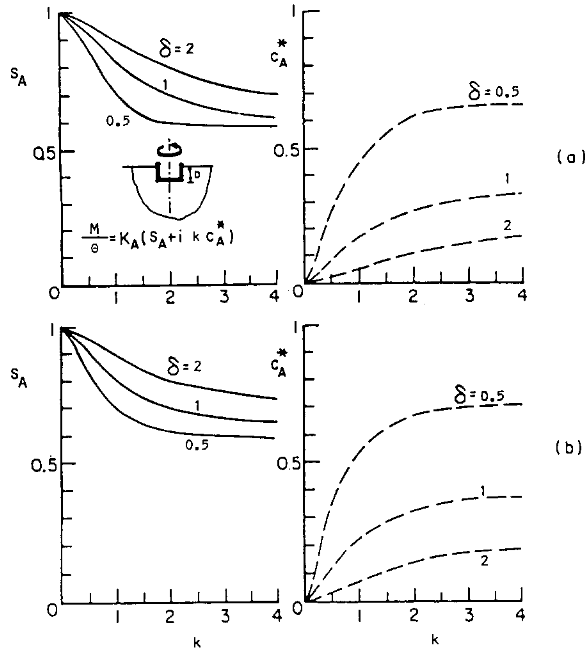

Fig. 6. Torsional dynamic stiffness and damping coefficients for a cylindrical embedded foundation for: (a) $D/a = 0.5$; and (b) $D/a = 1$.

in which: D = the depth of embedment below the free surface and $\chi = \chi(\delta D/a)$ is plotted in Fig. 5. The stiffness and damping coefficients are given in Fig. 6 as functions of k and δ for two values of embedment, $D = a/2$ and $D = a$.

MICHAEL C. CONSTANTINOU and GEORGE GAZETAS
Department of Civil Engineering
State University of New York at Buffalo
212 *Ketter Hall*
Buffalo
NY 14260
U.S.A.

REFERENCES

Constantinou, M. C. and Gazetas, G. (1984). Torsional vibration on anisotropic half-space. *J. Geotech. Engng, ASCE* **110**(11), 1549–1558.

Day, S. M. (1977). Finite-element analysis of seismic scattering problems. Ph.D. dissertation, University California at San Diego.

Luco, J. E. (1976). Torsion of a rigid cylinder embedded in an elastic half-space. *J. Appl. Mech.* No. 76-APM-FF, 419–423.

Luco, J. E. and Westmann, R. A. (1971). Dynamic response of circular footings. *J. Engng Mech. Div., ASCE* **97**(5), 1381–1395.

Tsai, Y. M. (1989). Torsional vibrations of a circular disk on an infinite transversely isotropic medium. *Int. J. Solids Structures* **25**(9), 1069–1076.